# Measuring the Polarization Rotation Angle of a Faraday Rotator Mirror with the Optical Vector Analyzer 

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## 1 Introduction

Faraday Rotator Mirrors (FRMs) utilize the Faraday Effect to rotate the input polarization state by some set amount, often by $90^{\circ}$. FRMs are commonly used as a component in isolators and are especially useful in fiber optic interferometers. If a Michelson interferometer is made with single mode fiber and a simple reflecting mirror (ie. a highreflectivity metal film), birefringence in either arm of the interferometer will cause the polarization state to rotate. When the light reflects off of the mirror and propagates back through the same path the optical retardance due to the fiber birefringence doubles. When the light from each arm of the interferometer is recombined, differences in the polarization orientation between the two arms will result in fading of the interferometer fringe amplitude visibility. Worse yet, this polarization based fading will typically be wavelength dependent and will be very sensitive to small changes in the input polarization state orientation. This visibility fading problem can be overcome by replacing the simple reflecting mirrors with FRMs set to rotate the polarization state by $90^{\circ}$. In this case, the retardance due to fiber birefringence on the path to and from the FRM is of equal magnitude but opposite sign and thus cancels, and the interferometer fringe visibility is optimized. An alternative is to construct a Michelson interferometer out of Polarization Maintaining (PM) components and maintain a stable input polarization state, but this alternative is typically more expensive and performs less well due to the variety of ways that optical power can leak from one polarization state to the other in a PM network. When using FRMs in an interferometer, however, if the FRM polarization state rotation is not exactly $90^{\circ}$ some fringe visibility fading may still occur. This application note details how Luna's Optical Vector Analyzer (OVA) can be used to easily deduce the error in the FRM rotation angle.

## 2 Measurement Set-Up

Although the OVA does not measure the absolute polarization angle of the device under test, it does compute the full Jones Matrix of the DUT over the scan range, and this

Jones Matrix can be analyzed to find the device Polarization Mode Dispersion (PMD) and Polarization Dependent Loss (PDL). If highly birefringent fiber, such as Polarization Maintaining fiber (PMF), is added to the path to the FRM, the magnitude of the PMD measured will be an indication of the rotation angle of the FRM.

To measure the polarization rotation angle of the test FRM, first the PMD of the PMF segment should be characterized, as depicted in Figure 1a. Leads on either side of the PMF segment may be Single Mode Fiber (SMF), and the quality of alignment of the connector keys to the stress rods of the PMF segment is inconsequential. The best way to get an accurate measurement of the PMD of the PMF segment is to terminate the PMF segment with a connector that will reflect strongly when unconnected. Any connector with a flat glass to air interface, such as an FC-PC connector, will work well. An angled connector would result in a weak reflection that may approach the OVA sensitivity level and result in a poor signal to noise ratio. If the PMF connector must be angled to match the FRM connector, then a short SMF patchcord to a highly reflective mirror could be used in place of the FRM, but any rotation of the polarization vector in the SMF to the mirror could lead to crossover from fast to slow modes of the PMF and thus reduce the apparent PMD associated with the PMF segment. After the PMF segment is scanned and the PMD recorded, the test FRM is connected downstream of the PMF segment as indicated in Figure 1b, and the PMD trace for the FRM is recorded.


Figure 1. Test set-up with PM fiber segment in between OVA and mirror under test, in reflection mode.

## 3 Jones Matrix Calculation of PMD

Consider the first case in which the mirror in Figure 1 is a simple reflector. Neglecting birefringence contributions from the SMF segments and connectors, and assuming a perfect reflector, the total Jones Matrix $\mathrm{J}_{\text {TOT }}$ measured by the OVA will be:

$$
J_{\text {TOT }}=J_{P M} \cdot M_{R} \cdot J_{P M} .
$$

The element matrices are:

$$
J_{P M}=\left[\begin{array}{cc}
e^{i \omega \tau_{P M} / 2} & 0 \\
0 & e^{-i \omega \tau_{P M} / 2}
\end{array}\right], M_{R}=\left[\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right] .
$$

The result for $\mathrm{J}_{\text {TOT }}$ is:

$$
J_{\text {ТОT }}=\left[\begin{array}{cc}
e^{i \omega \tau_{P M}} & 0  \tag{3}\\
0 & e^{-i \omega \tau_{P M}}
\end{array}\right] .
$$

In equations 2(a) and 3 above, $\omega$ is the angular optical frequency and $\tau_{\text {PM }}$ is the single pass optical delay between the fast and slow modes of the PM fiber segment.

Polarization mode dispersion (PMD), or differential group delay (DGD), is the maximum difference in group delay over all polarization states in the optical time domain.
Alternatively, PMD is also the maximum rate of change of the polarization state in the spectral domain, as described in Chapter 8 of the OVA User Guide. To calculate the PMD, we find the eigenvalues $\lambda_{1}$ and $\lambda_{2}$ of the matrix $\mathrm{J}_{\text {TOT }}(\omega+\Delta \omega) \cdot \mathrm{J}_{\text {TOT }}(\omega)^{-1}$, and compute:

$$
\begin{equation*}
P M D=\left|\angle\left(\lambda_{1} / \lambda_{2}\right) \frac{1}{\Delta \omega}\right| . \tag{4}
\end{equation*}
$$

The sign $\angle$ denotes the phase angle of the complex ratio.
For $\mathrm{J}_{\text {TOT }}$ in Equation 3, we find the following results for the eigenvalues $\lambda_{1}$ and $\lambda_{2}$ and for the PMD for the simple reflector $\mathrm{PMD}_{\mathrm{R}}$ :

$$
\lambda_{1}=e^{i \Delta \omega \tau_{P M}}, \quad \lambda_{2}=e^{-i \Delta \omega \tau_{P M}}, \quad \text { and } P M D_{R}=2 \tau_{P M} .
$$

If we were to transform the matrix $\mathrm{J}_{\text {тот }}$ in Equation 3 into the time domain and take the eigenvalues, the eigenvalues would be represented by two peaks separated by $2 \tau_{\text {Рм }}$, so the two different methods of obtaining the PMD are clearly equivalent in this simple case.

In the case in which the mirror is a perfect FRM which rotates the input state by exactly $90^{\circ}$, the mirror Jones Matrix becomes:

$$
M_{F R M, \text { perfect }}=\left[\begin{array}{cc}
0 & 1  \tag{6}\\
-1 & 0
\end{array}\right] .
$$

The result for $\mathrm{J}_{\text {TOT }}$ is:

$$
J_{\text {ТОТ }}=\left[\begin{array}{cc}
0 & 1  \tag{7}\\
-1 & 0
\end{array}\right] .
$$

In this case there is no delay between the eigenvalues in the time domain, and the eigenvalues do not evolve in the spectral domain, so the resulting PMD is zero:

$$
\begin{equation*}
\lambda_{1}=\lambda_{2}=1, \quad P M D_{F R M, \text { perfect }}=0 . \tag{8}
\end{equation*}
$$

Consider the case where instead of a $90^{\circ}$ rotation, the real rotation is $90^{\circ}+\varepsilon$, where $\varepsilon$ is some small angle, $\varepsilon \ll 90^{\circ}$. The mirror Jones Matrix becomes:

$$
M_{F R M}=\left[\begin{array}{cc}
-\sin (\varepsilon) & \cos (\varepsilon)  \tag{9}\\
-\cos (\varepsilon) & -\sin (\varepsilon)
\end{array}\right] .
$$

The result for $\mathrm{J}_{\text {TOT }}$ is:

$$
J_{\text {Тот }}=\left[\begin{array}{cc}
-\sin (\varepsilon) e^{i \omega \tau_{P M}} & \cos (\varepsilon)  \tag{10}\\
-\cos (\varepsilon) & -\sin (\varepsilon) e^{-i \omega \tau_{P M}}
\end{array}\right] .
$$

In this case the eigenvalues are substantially more work to calculate:

$$
\begin{equation*}
\lambda_{1,2}=x \pm i \sqrt{1-x^{2}}, \text { with } x=\cos ^{2}(\varepsilon)+\sin ^{2}(\varepsilon) \cos \left(\Delta \omega \tau_{P M}\right) \tag{11}
\end{equation*}
$$

If we make small angle approximations for $\Delta \omega \tau_{P M}$ (this term in practice would be $\ll$ 0.001 radian), the end result for PMD simplifies considerably:

$$
\begin{equation*}
P M D_{F R M}=\left|2 \sin (\varepsilon) \tau_{P M}\right| \tag{12}
\end{equation*}
$$

In this case the time domain representation of the impulse response of the DUT would have a strong center peak with side lobes that are only $\sin (\varepsilon)$ as high and separated from the center peak by $\pm \tau_{\text {PM. }}$. The center peak of the impulse response contributes nothing to PMD, and the two side lobes would each contribute roughly $\sin (\varepsilon) \tau_{\mathrm{PM}}$ to PMD. Thus the error angle of the FRM can be calculated from the ratio of the PMD of the PMF segment with the FRM attached, and of the PMF segment alone, respectively:

$$
\begin{equation*}
|\varepsilon|=\sin ^{-1}\left(P M D_{F R M, \text { max }} / P M D_{R}\right) . \tag{13}
\end{equation*}
$$

## 4 Measurement Example

Figure 2 shows time domain amplitude plots for a 10 m PMF segment terminated with a FC-PC connector, and for a test FRM connected to the end of the PMF segment, as depicted in Figure 1. Both have a center lobe with two side lobes separated by the same extent, but for the PMF segment alone the center lobe is low compared to the side lobes, and the opposite is true for the FRM. The low center peak in Figure 2 a is an indication that very little light has crossed over from the fast axis to the slow axis (and vice-versa) of the PMF segment at the connector face. If the center peak is large
compared to either of the side peaks, the user should clean or re-polish the connector end face. Cursors indicate the separation of the side peaks are 22.6 ps , and this is the value for the PMD we expect to measure for the PMF segment. The side peaks of the scan of the FRM in Figure 2b have the same separation, but are much smaller in proportion to the center peak, and we expect the PMD for this case to be much smaller because most of the power in the time impulse response is in the center peak. Note that the time domain filter limits must be placed outside of these side lobe locations in order to get an accurate PMD measurement. For both of these OVA scans, the time domain filters were set at a width of approximately 47 ps to give Window Resolution Bandwidth of 100 pm, and the spectral Filter Resolution Bandwidth was set to 0 so that no additional spectral filtering was applied.


Figure 2. Time domain amplitude profiles of a) a FC-PC connector at the end of a 10 m PMF segment and b) a FRM with a 10 m PMF segment in the path to the OVA.

Figure 3 shows the PMD plots for the two cases described in Figure 2. The scan of the PMF segment alone shows high levels of PMD, with an average of about 22.5 ps , and the scan of the FRM shows much lower average PMD, between 0 and 2.7 ps over the scan range.


Figure 3. PMD for a ) the 10 m PMF segment alone, and b) for the FRM with the 10 m PMF segment in series.

Figure 4 shows the result for FRM rotation error angle $|\varepsilon|$ calculated using Equation 13.


Figure 4. FRM polarization rotation angle error, as computed from the PMD curves in Figure 3.

## 5 Summary

This method of calculating the FRM polarization rotation error angle from OVA PMD measurements is rapid as it only requires scanning the FRM with a PMF lead and recording the PMD for the wavelength range of interest. No alignment of the input polarization state or of an analyzing polarizer is necessary. The PMD of the PM fiber lead only needs to be measured once. No particular connector key alignment to the PMF stress rods is necessary.

This minimum resolvable rotation error angle will be dependent either on the PMD instrument accuracy ( 0.08 ps max, with a 30 pm time domain window and 64 averages), or by the PMD inherent to the FRM. Since the PMD of the PMF segment and the FRM mirror have no particular defined orientation with respect to each other, they may arbitrarily add or subtract from each other, so a key accuracy requirement is that the PMD of the PMF segment is much larger than the FRM PMD. Since PMD of the PMF segment alone is in the denominator of the calculation, better rotation error angle resolution is obtained by using a longer PMF segment. However at some point the polarization crossover in long lengths of PMF would start to reduce accuracy. A PMF segment length of 10 m appears to be ample to achieve resolution of close to 0.1 degree for a typical FRM, and the PMD of such a segment length greatly exceeds the expected PMD of the FRM.

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